Multistage Fusion Reaction Rates In An Adiabatically Compressed Plasma

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Dedication: Lloyd Motz

(1910 - 2004)

Professor: Columbia University

Motz suggested:

In stars, gravity is driver for fusion..

Consider analogous case:

Compression by a piston.

Nuclear fusion rates $\approx n^2$

n = particle density

Magnetic confinement methods:

Thin plasmas: $n \approx 10^{15}$ cm^3

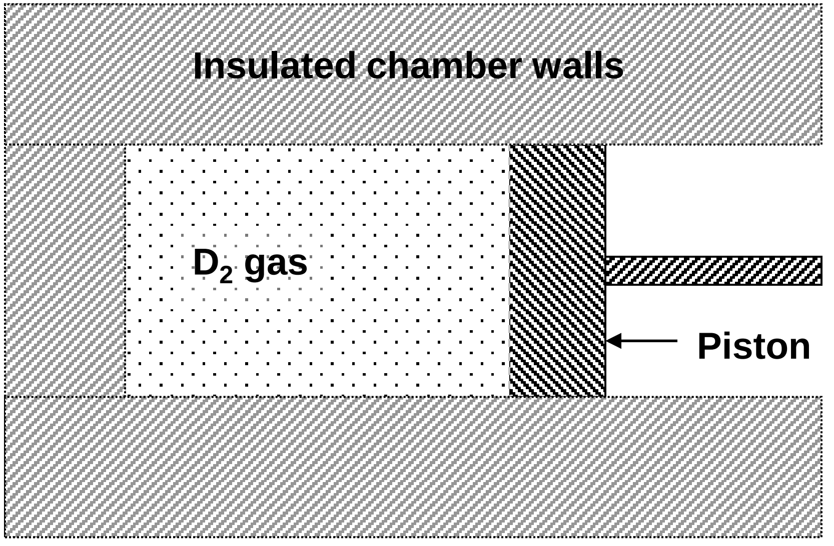
$\Rightarrow$ Require $T \approx 10^8$ K

<u>Propose</u>

Method to exploit n^2 factor

$\Rightarrow$ interesting fusion rates at lower T

Mechanical adiabatic compression



- Dense gas of D_2 undergoes adiabatic compression
  - Rapid process - - explosively driven
- Adiabatically insulated chamber – retain energy
- Operate at reduced degrees of freedom

- Starting conditions

One mole D_2 at atmospheric pressure and room temperature: $T_0 = 300$ K;

$V_0 = 2.46\times10^4$ cm^3

$n_0 = 2N_A / V_0 = 4.90 \times 10^{19}$ cm^{-3}

N_A = Avogadro's no.

Factor of 2: Have 2 atoms/molecule

Apply compression

T increases

D_2 molecules dissociate $\Rightarrow$ D atoms

D atoms ionize $\Rightarrow$ deuteron-electron plasma

$\Rightarrow$ Fusion of deuterons

CAVEATS

Will make simplifying assumptions

May neglect important effects

Compensate with conservative estimates

Assumptions

Reversible adiabatic compression

Apply equilibrium thermodynamics

Can reduce degrees of freedom of gas

Degrees of freedom

- Specific heat ratio γ related to number of degrees of freedom f of the gas:
  - $\gamma = (f + 2) / f$

- For monoatomic gas: $f = 3$

If deprive particles of some freedom of motion
$\Rightarrow$ Larger T increase for given energy input.

Accomplish by:
(1) External magnetic field(s)
(2) Electric discharge in direction of piston motion
Also $\Rightarrow$ Pinch Effect

Previous Work: ICENES 2007

Assumed fusion to occur only at end of compression. Single step process.

Perfect gas and Van der Waals gas

Best conditions $\Rightarrow$ energy breakeven.

Better for VdW gas.

Present work: Multistep process

Compute at various stages of compression:

Apply compression; compute work done and T increase;

At new T compute reaction rate, fusion energy release and no. particles reacted;

Compute new T corresponding to energy released and new particle density;

Apply compression to new volume, etc.

Van der Waals Equation of State

$$\left(P + a\frac{N^2}{V^2}\right)(V - Nb) = NRT$$

N = number of moles

Pressure correction: intermolecular forces

Volume correction: finite size of molecules

Limits compression factor: $\beta < V_0 / Nb$

Take H values for *a*, *b*: $b \approx 0.027$ L-mol^{-1}

Work to compress gas

$$W = -\int_{V_0}^{V} P\,dV$$

Solve for *P* from equation of state.

Introduce compression ratio: $\beta = V_0 / V$

Substitute β and γ-1 = 2/*f* :

Work to compress van der Waals gas

$$W = \frac{NRT_0 f \beta^{2/f}}{2}\left[\left(\frac{V_0 - Nb}{V_0 - \beta Nb}\right)^{2/f} - \beta^{-2/f}\right] - \frac{aN^2}{V_0}(\beta - 1)$$

$$= W_{ideal}\left[\left(\frac{V_0 - Nb}{V_0 - \beta Nb}\right)^{2/f} - \beta^{-2/f}\right] - \frac{aN^2}{V_0}(\beta - 1)$$

Adiabatic temperature increase for van der Waals gas

$$T = T_0 \left(\frac{V_0 - Nb}{V - Nb} \right)^{\gamma - 1}$$

$$= T_0 \beta^{2/f} \left(\frac{V_0 - Nb}{V_0 - \beta Nb} \right)^{2/f}$$

$$= T_{ideal} \left(\frac{V_0 - Nb}{V_0 - \beta Nb} \right)^{2/f}$$

- <u>Energy Release</u>

Reaction rate: $r = \frac{1}{2} n^2 <\sigma v>$

σ = reaction cross section

v = relative velocity of interacting nuclei

Energy release in time δt: $\delta E = r\, Q\, V\, \delta t$

Q = average energy release/reaction

V = final volume

Primary reactions:

$D + D \rightarrow T + p + 4.03$ MeV

$D + D \rightarrow {}^3He + n + 3.27$ MeV

$<\sigma v> \approx 10^{-14}\, T^{-2/3} \exp(-19T^{-1/3})\ cm^3 / s$, $T < 50$ keV

Secondary reactions

- $D + T \rightarrow \alpha + n + 17.6$ MeV
- $D + {}^3He \rightarrow \alpha + p + 18.3$ MeV

Simplification

- Consider only D+D: Average Q = 3.65 MeV

Fuel burnup fraction

Fraction of fuel consumed in volume *V* and time interval δt:

$$\frac{rV\,\delta t}{n_0 V_0} = \frac{r\,\delta t}{n_0 \beta}$$

Multistep Calculation

Divide entire compression into *S* steps.

Compression ratio for step *j:*

$$\beta_j = \frac{j\beta}{S}$$

Previous work: δt = 0.001 sec

Present work: $\delta t = 0.001 S^{-1}$ sec

Software: CHEMKIN II

Table 1: One step values for a vdW gas.

f	β	T_1 (K)	r (cm^{-3}-s^{-1})	rQ *(MeV/cc/s)*	$\delta E/W$	$r\delta t/\beta n_0$
3	100	8×10^{3}	6×10^{-63}	$\sim 10^{-62}$	2×10^{-80}	1×10^{-87}
3	200	2×10^{4}	2×10^{-43}	$\sim 10^{-43}$	1×10^{-61}	2×10^{-68}
2	100	4×10^{4}	3×10^{-24}	$\sim 10^{-24}$	1×10^{-42}	5×10^{-49}
2	200	1×10^{5}	3×10^{-8}	$\sim 10^{-7}$	3×10^{-27}	3×10^{-33}
1	100	5×10^{6}	7×10^{18}	$\sim 10^{19}$	2×10^{-2}	1×10^{-6}
1	200	4×10^{7}	2×10^{24}	$\sim 10^{25}$	4×10^{2}	2×10^{-1}

T_1 is the temperature at the end of the compression step, before the fusion reaction occurs. Reaction rate (r), power density(rQ), the ratio $\delta E/W$ and the fuel burnup fraction($r\delta t/\beta n_0$) are computed for a time interval of 0.001 seconds and for various values of compression factor and degrees of freedom in a van der Waals gas.

Table 2: Multi-step compression with f =1 and various β.

β	T_1 (K)	r (cm^{-3}-s^{-1})	rQ (MeV/cc/s)	δE/W	rδt/bn_o	Heat (J)	ΔT_{fusion} (K)	T_2 (K)
1	3×10^2							3×10^2
50	9×10^5	6×10^{10}	2×10^{11}	5×10^{-10}	6×10^{-15}	4×10^{-3}	5×10^{-4}	9×10^5
100	5×10^6	7×10^{18}	3×10^{19}	6×10^{-3}	4×10^{-7}	3×10^5	3×10^4	5×10^6
150	2×10^7	2×10^{22}	6×10^{22}	3	6×10^{-4}	4×10^8	5×10^7	7×10^7
200	2×10^8	9×10^{25}	3×10^{26}	5×10^3	1	2×10^{12}	7×10^{11}	7×10^{11}

4-Step compression, each step compressed instantaneously to T_1 and held for 0.25 msec. "Heat" is the energy released by fusion during 0.25 msec and T_2 is the temperature after adding ΔT_{fusion}, the energy released, to T_1. The next value for T_1 is calculated by compressing the gas to the next β–value from the previous T_2.

Applications

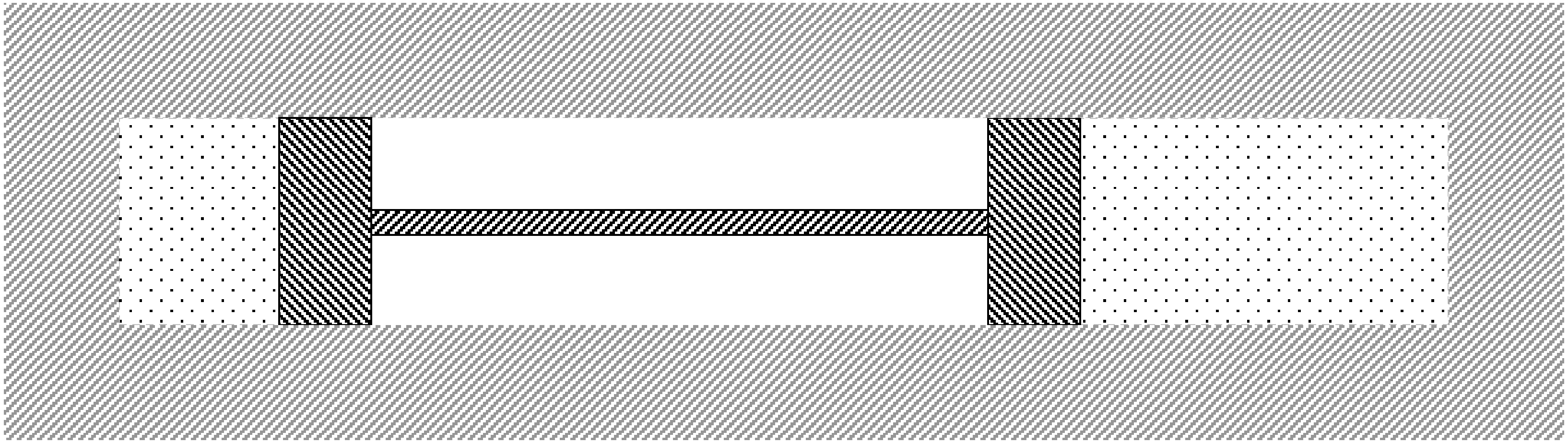
Single compression

Neutron source to initiate fission ?

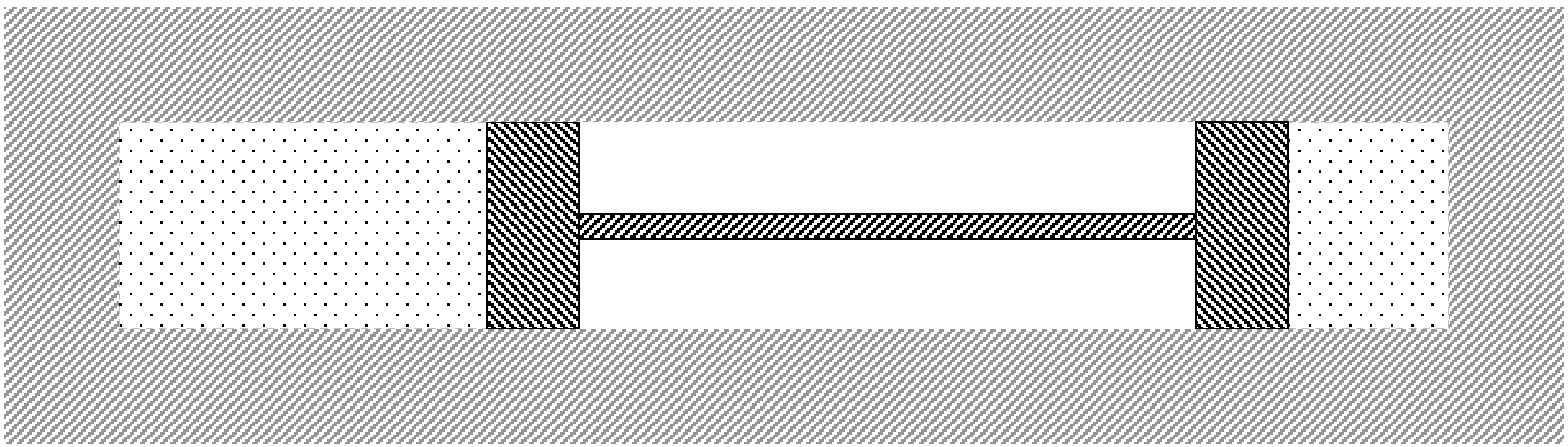
Multiple compressions - dual chambers

Reciprocating device

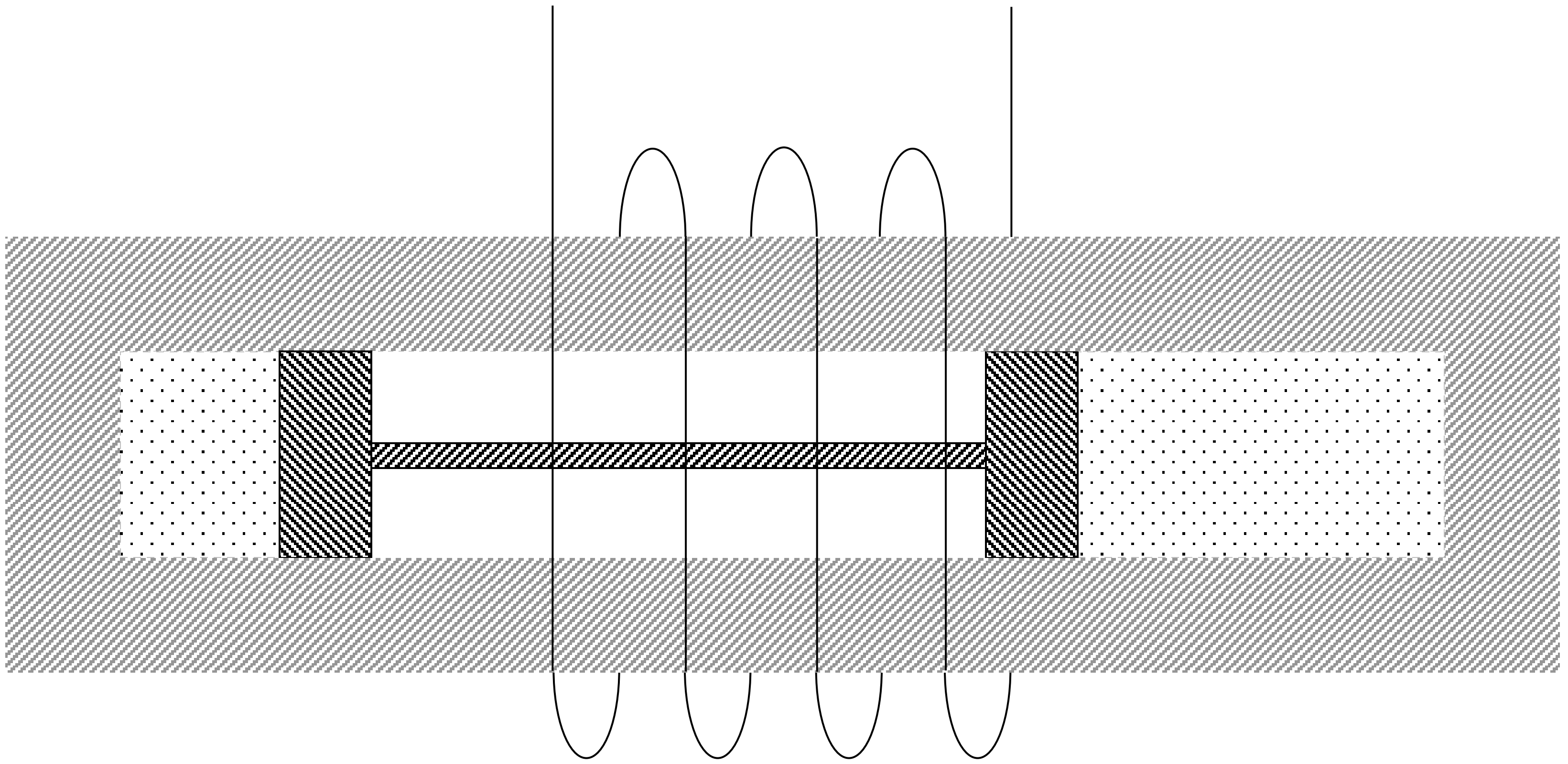
Dual Pistons



Dual Pistons



Dual Pistons with Coil to Extract Work



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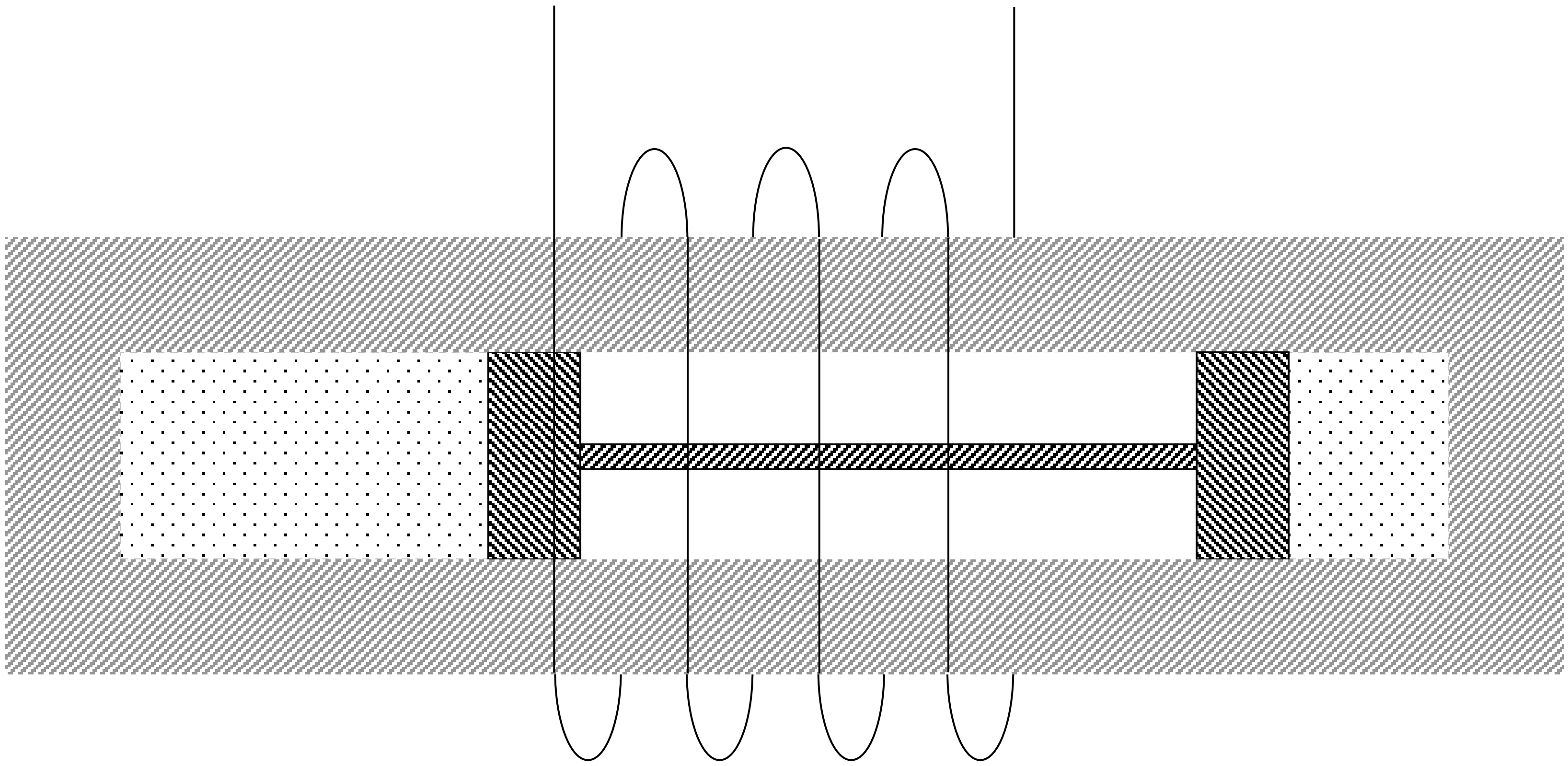

Summary and Conclusions

- Exploited n^2 factor and reduced degrees of freedom
- Adiabatic conditions $\Rightarrow$ energy retained internally
- Found some favorable cases
- To be more realistic:
  - Not all the input energy serves to compress gas
  - Take into account work needed to establish fields
  - Consider particle losses via leakage

TrIed to compensate by ignoring:

- D-T reactions (more energetic than D-D)
- Volume compression from Pinch Effect
- Possible enhancement factors
  - Coat walls with deuterides to increase n
  - Screening effects of electrons

Although analysis was performed under the simplest assumptions, some favorable results suggest deeper study is warranted.

Should consider: Other equations of state

Implementation

Requires ingenuity to overcome technical challenges.

HOPE SOMEONE CAN TRY IT!!