THEORETICAL MODELING OF RADIAL STANDING WAVE REACTOR

Xue-Nong Chen, Dalin Zhang and Werner Maschek

Institute for Nuclear and Energy Technologies, KIT, Germany

KARLSRUHE INSTITUTE OF TECHNOLOGY (KIT), INSTITUT für KERN UND ENERGETIK (IKET)

ICENES2011, San Francisco, May 15-19, 2011

❑ Introduction

➢What is Traveling Wave Reactor (TWR) or Candle?

It is actually a fast breeder reactor, which employs breeding/burning wave mechanism, so that it can use the natural or depleted uranium.

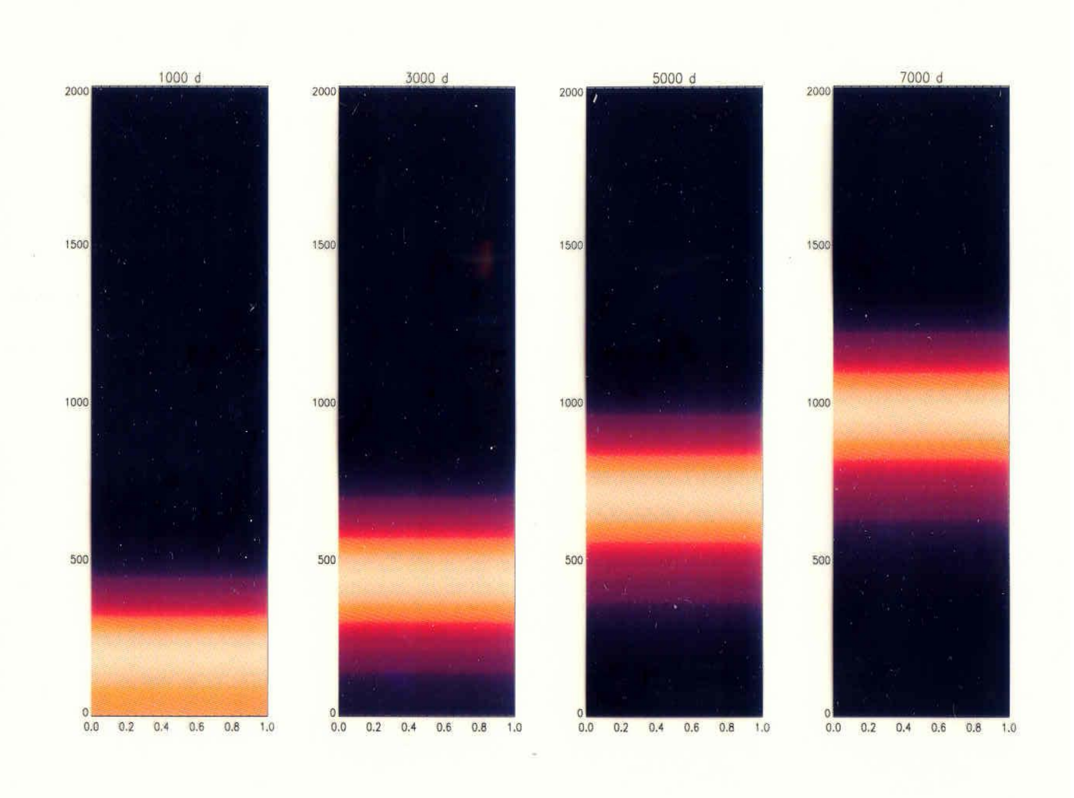
➢Why is TWR interesting (attractive)?

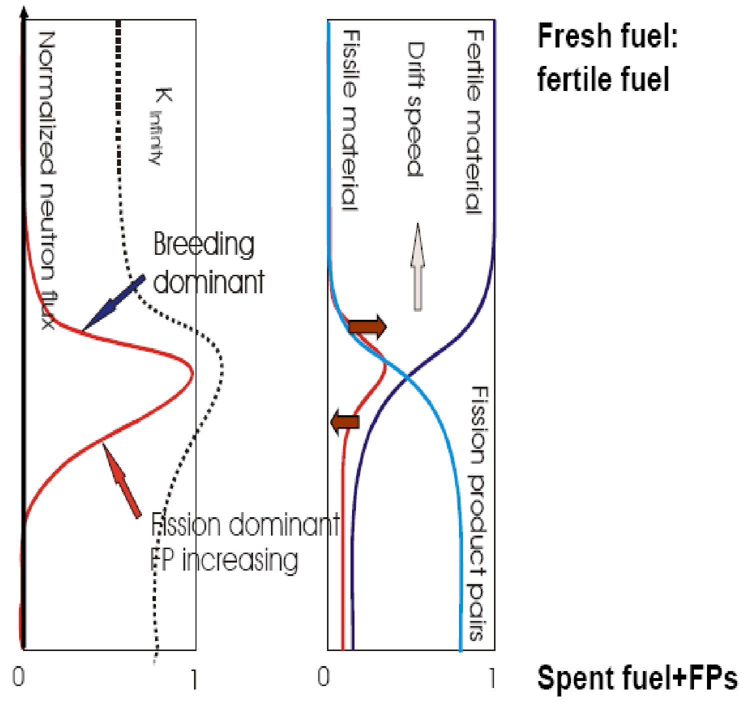
Extremely high fuel utilization, due to high burn-up and non-enrichment

➢TerraPower made designs of such reactors based on sodium cooled reactor technology.

❑ Introduction

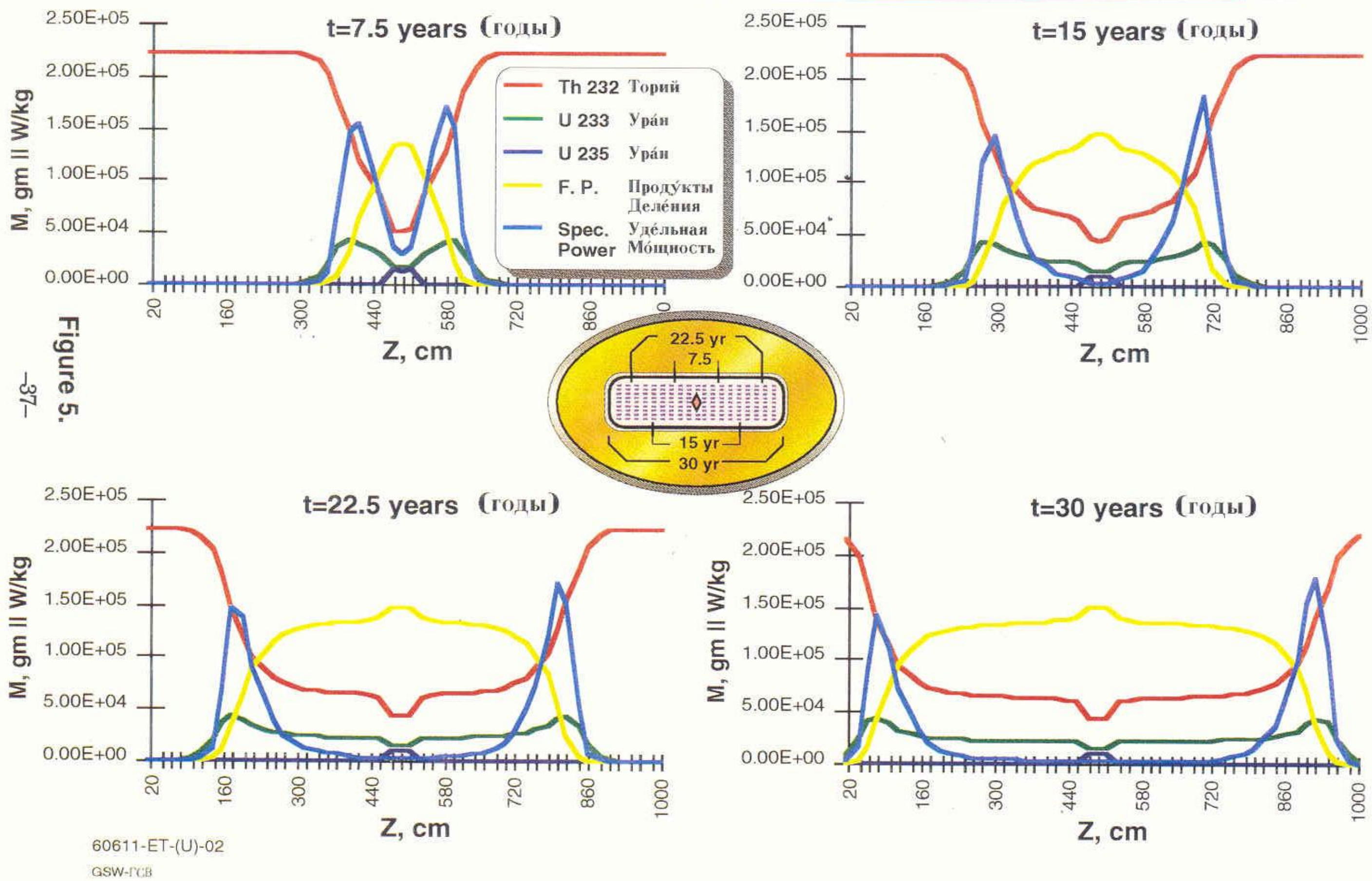
➢Traveling wave reactor mechanism





➢History of this concept, see Wikipedia Traveling Wave Reactor

Salient Features Of Nuclear Deflagration Wave Propagation (Full–Power Case)



E. Teller *et al*.: ICENES’96, Obninsk,

Sekimoto, van Dam, ICENES’00, Petten

❑ Introduction

- Theoretically, it is straightforward to understand such traveling wave in a plane geometry. This may be the reason for why most of studies in the past were on the axial traveling wave.
- BUT practically, how to realize this mechanism is a big question. There are two points here:

1. Since the core is finite, instead of the power shape moving, we have to consider fuel moving.
2. We have to consider the radial fuel shuffling, since it is a today's standard technology.

It is easy to imagine that the radial fuel SA shuffling can generate a continuous fuel motion, if it is done in the order of radius.

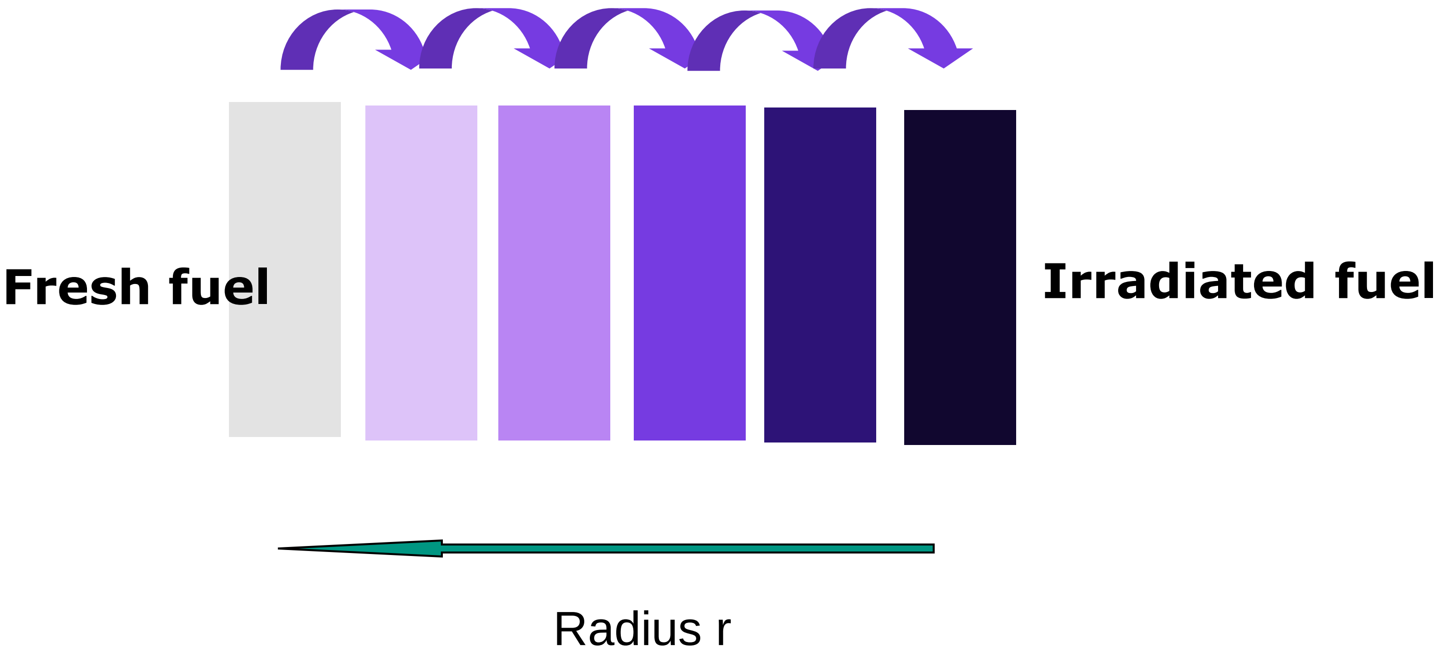
The question arising here: Can a standing breeding/burning wave exist under this radial fuel shuffling.

- This paper deals with radial fuel shuffling and provides a simple modeling and its numerical solution for the asymptotic state in R-Z geometry.

❑ Mathematical Modeling

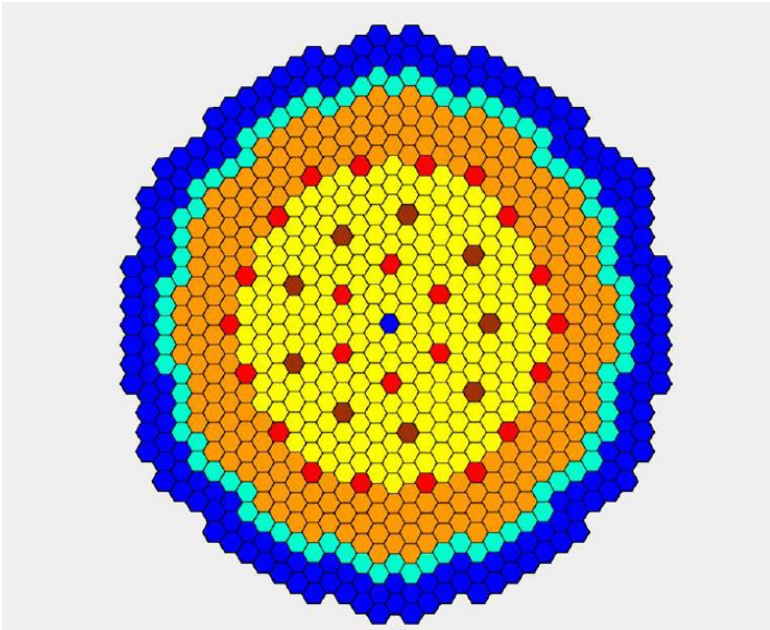
Karlsruhe Institute of Technology

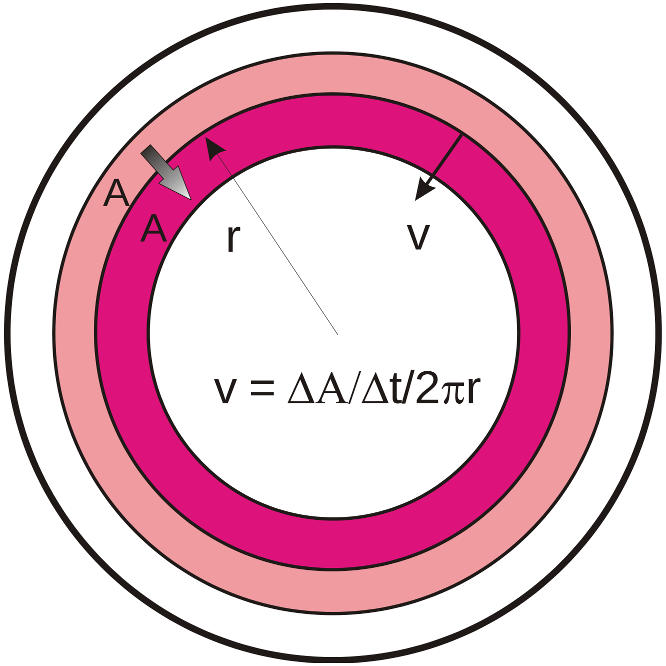
- **Radial fuel shuffling in the order of radius (e.g. inward)**



❑ Mathematical Modeling

- **Radial continuous fuel shuffling model (e.g. inward)**





$$A' = \frac{\Delta A}{\Delta t}$$

Discrete, but in order of radius

$$v(r) = \frac{A'}{2\pi\, r}$$

Continuous, but drifting speed not uniform

❑ Mathematical Modeling

KIT Karlsruhe Institute of Technology

➢ **1-Group Diffusion Model Coupled with Burn-up Equations**

$$\nabla\left(D \cdot \nabla \phi\right) + \left(\nu\Sigma_f - \Sigma_a\right)\phi = 0$$

$$\Sigma_a = \sum_n N_n \sigma_{a,n}, \quad \nu\Sigma_f = \sum_n N_n \nu_n \sigma_{f,n}, \quad \Sigma_{tr} = \sum_n N_n \sigma_{tr,n}, \quad D = \frac{1}{3\Sigma_{tr}}$$

$$\psi = \int_0^t \phi \, dt$$

ICENES2011 San Francisco

U-Pu conversion cycle

$$^{238}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}\mathrm{U} \xrightarrow[23.5\mathrm{m}]{\beta^-} {}^{239}\mathrm{Np} \xrightarrow[2.35\mathrm{d}]{\beta^-} {}^{239}\mathrm{Pu}$$

$$^{239}\mathrm{Pu} + \mathrm{n} \rightarrow {}^{240}\mathrm{Pu} \xrightarrow[6.5\times10^3\,y]{\alpha} {}^{236}\mathrm{U}$$

$$^{240}\mathrm{Pu} + \mathrm{n} \rightarrow {}^{241}\mathrm{Pu} \xrightarrow[15y]{\beta^-} {}^{241}\mathrm{Am}$$

●●●●●●

Th-U conversion cycle

$$^{232}\mathrm{Th} + \mathrm{n} \rightarrow {}^{233}\mathrm{Th} \xrightarrow[22.3\mathrm{m}]{\beta^-} {}^{233}\mathrm{Pa} \xrightarrow[27.0\mathrm{d}]{\beta^-} {}^{233}\mathrm{U}$$

$$^{233}\mathrm{U} + \mathrm{n} \rightarrow {}^{234}\mathrm{U}$$

$$^{234}\mathrm{U} + \mathrm{n} \rightarrow {}^{235}\mathrm{U}$$

fertile **fissile**

❑ Mathematical Modeling

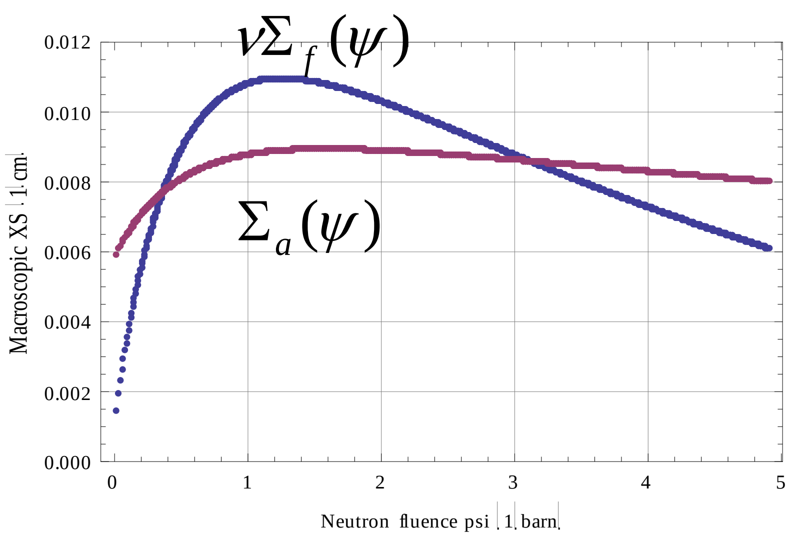
➢ **Burn-up model:**

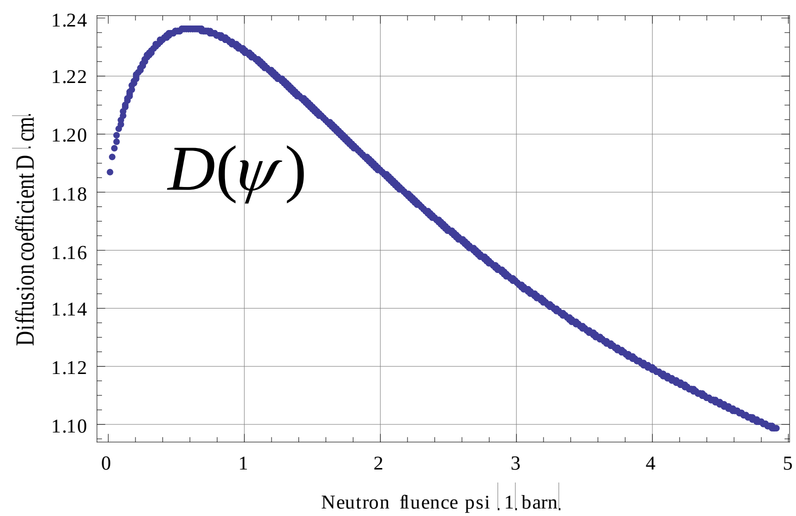
U-Pu conversion chosen as an example

If Radioactive decay processes are neglected, the nuclide atom number densities are only functions of neutron fluence

➢ **The macroscopic XSs are provided by ERANOS burn-up cell calculation under following conditions (see Zhang et al. 2011):**

Metallic fuel: 50 vol% Na coolant: 30 vol% Steel: 20 vol%



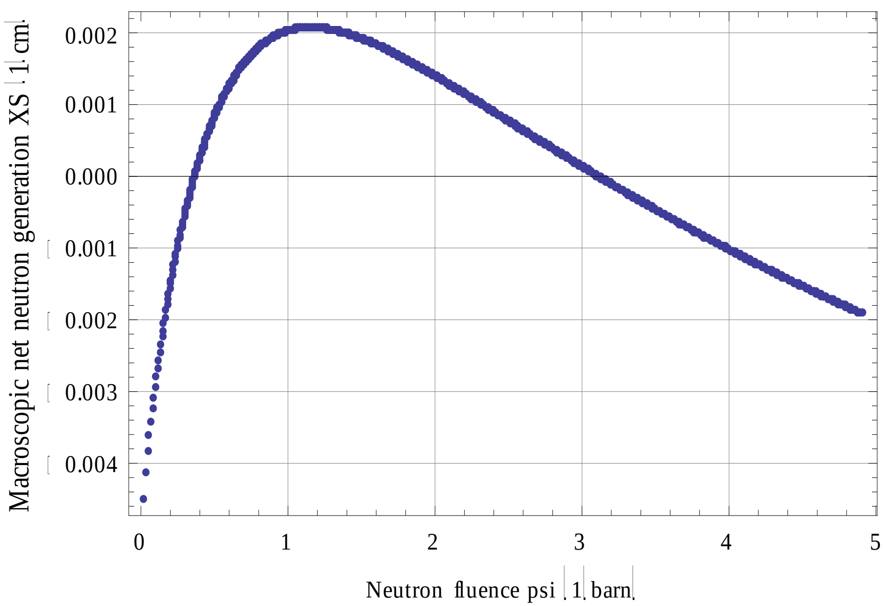


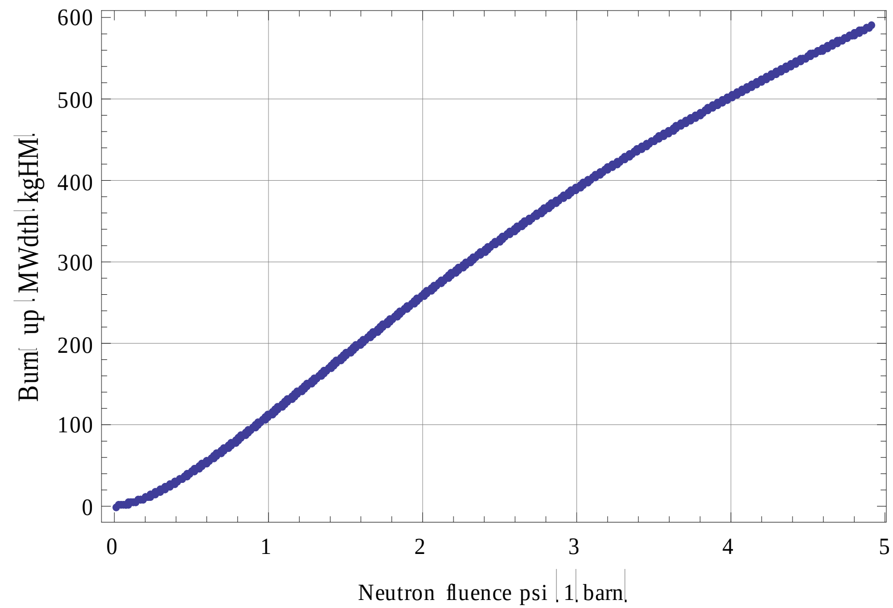
❑ Mathematical Modeling

➢ **The net neutron generation XS and burnup as functions of fluence**

$$F(\psi) = \nu\Sigma_f - \Sigma_a$$

$$Burnup(\psi)$$





❑ Mathematical Modeling

➢ **Radial fuel shuffling (inwards)**

$$d\psi = \phi dt \text{ and } dt = -\frac{dr}{v} \text{ with } v = \frac{A'}{2\pi r} \quad \Rightarrow \quad \frac{\partial \psi}{\partial r} = -\frac{2\pi r}{A'}\phi$$

➢ **Boundary conditions, which are trivial at the extrapolated boundaries**

$$\psi = 0 \text{ and } \phi = 0 \text{ at } r = R$$

$$\frac{\partial \phi}{\partial r} = 0 \text{ at } r = r_0$$

$$\phi = 0 \text{ at } x = 0 \text{ and } L$$

❑ Mathematical Modeling

➢ **Asymptotical eigenvalue problem**

$$\frac{1}{r}\frac{\partial}{\partial r}\left(D(\psi)\, r\frac{\partial}{\partial r}\phi\right)+\frac{\partial}{\partial z}\left(D(\psi)\frac{\partial}{\partial z}\phi\right)+\underbrace{\left(\frac{\nu\Sigma_f(\psi)}{k_{eff}}-\Sigma_a(\psi)\right)}_{F_{k_{eff}}(\psi)}\phi=0$$

$$\frac{d\psi}{dr}=-\frac{2\pi r}{A'}\phi$$

➢ **Remarks:**

k_{eff} is a measure of criticality for the asymptotic state

This eigenvalue problem is nonlinear, which is different from the conventional linear eigenvalue problem.

For both 1-D plane and cylindrical geometries there is the so-called fundamental burn-up mode, see Chen et al. (2005, 2008, 2011)

❑ Integration and Normalization

- **Normalization with** $v_R = \dfrac{A'}{2\pi R}$ and $\phi_{\max}$

$$\sigma_{v_R} = \frac{v_R}{R\phi_{\max}}$$ [barn] $$\sigma_{v_R} N_{HM,0} \quad \text{for } \Sigma$$ $$\text{e.g. } \bar{D} = \frac{D\phi_{\max}}{v_R N_{HM,0} R}$$

$$\bar{\phi} = \frac{\phi}{\phi_{\max}} \qquad \bar{\psi} = \sigma_{v_R}\psi \qquad \bar{r} = \frac{r}{R}$$

- **Normalized Equations**

$$\bar{\nabla}\left(\bar{D}(\bar{\psi})\,\bar{\nabla}\bar{\phi}\right) + \left(\frac{\nu\bar{\Sigma}_f(\bar{\psi})}{k_{eff}} - \bar{\Sigma}_a(\bar{\psi})\right)\bar{\phi} = 0 \qquad \frac{d\bar{\psi}}{d\bar{r}} = -\bar{r}\bar{\phi}$$

$$\bar{\psi} = 0 \text{ and } \bar{\phi} = 0 \text{ at } \bar{r} = 1 \qquad \frac{d\bar{\phi}}{d\bar{r}} = 0 \text{ at } \bar{r} = 0 \qquad \bar{\phi} = 0 \text{ at } z = 0 \text{ and } L$$

❑Normalization and Iteration

- **Iteration**

$$\nu\overline{\Sigma}_f(\overline{\psi}^{(n)})\frac{\partial\overline{\phi}^{(n+1)}}{\partial\tau}=\overline{\nabla}\left(\overline{D}(\overline{\psi}^{(n)})\cdot\overline{\nabla}\overline{\phi}^{(n+1)}\right)+\left(\frac{\nu\overline{\Sigma}_f(\overline{\psi}^{(n)})}{k_{eff}^{(n)}}-\overline{\Sigma}_a(\overline{\psi}^{(n)})\right)\overline{\phi}^{(n+1)}$$

$$\frac{\partial\overline{\psi}^{(n+1)}}{\partial\overline{r}}=-\overline{r}\overline{\phi}^{(n+1)}(\tau=T^{(n+1)})$$

$$\tau \text{ from } \tau^{(n)} \text{ to } \tau^{(n+1)}$$

The problem is solved by Mathematica. The solution will be convergent after several iterations, e.g. relative error less than 0.5% after 10 times.

Solution and Results

- **Parameters used in the example**

Parameter	R	r_0	L	D_0	$N_{HM,0}$	ϕ_{max}
Value	150 cm	0 cm	200 cm	1.184 cm	$2.13\ 10^{22}\ cm^{-3}$	$3\ 10^{15}\ cm^{-2}\ s^{-1}$

- **Results**

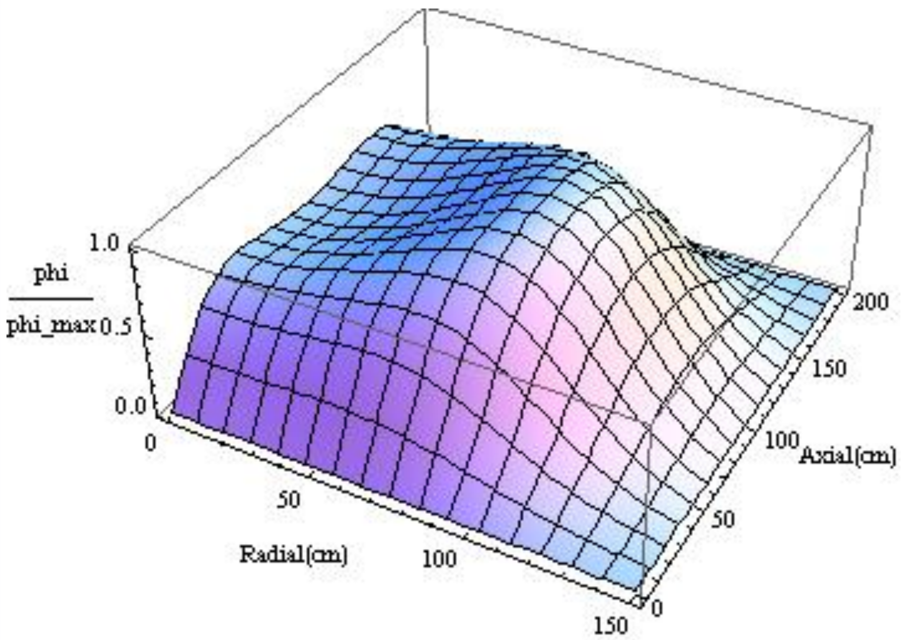
Variable	v_R / ϕ_{max}	k_{eff}	v_R	Peaking factor	*Av. burn-up*	*Max. burn-up*
Value	**11.70**	**1.0223**	**1.1064**	**2.423**	**346**	**488**
Dimension	barn cm		cm/year		GWdth/ton	GWdth/ton

The total fuel irradiation time is about 68 years

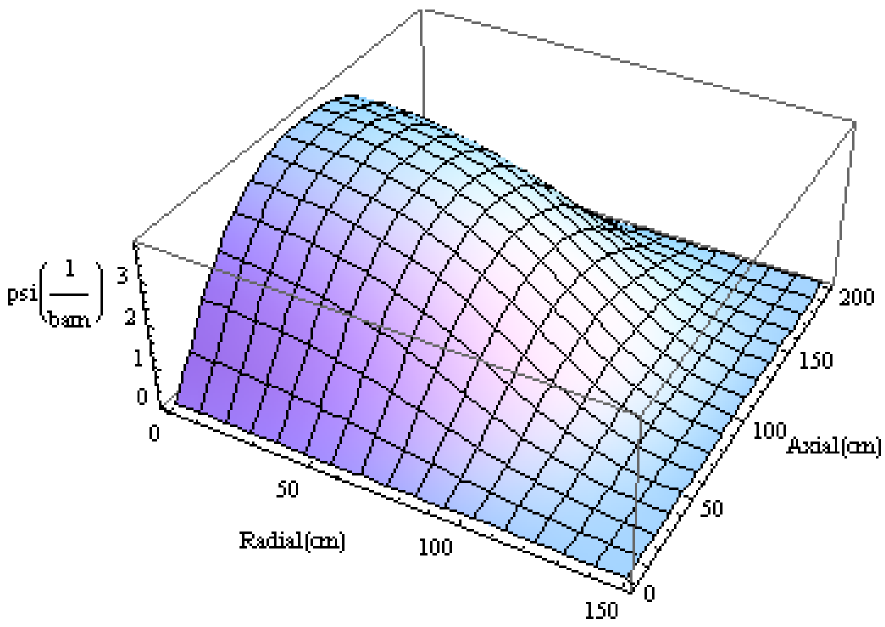
❑ Solution and Results

➢ **Radial inward fuel shuffling**

Neutron Flux



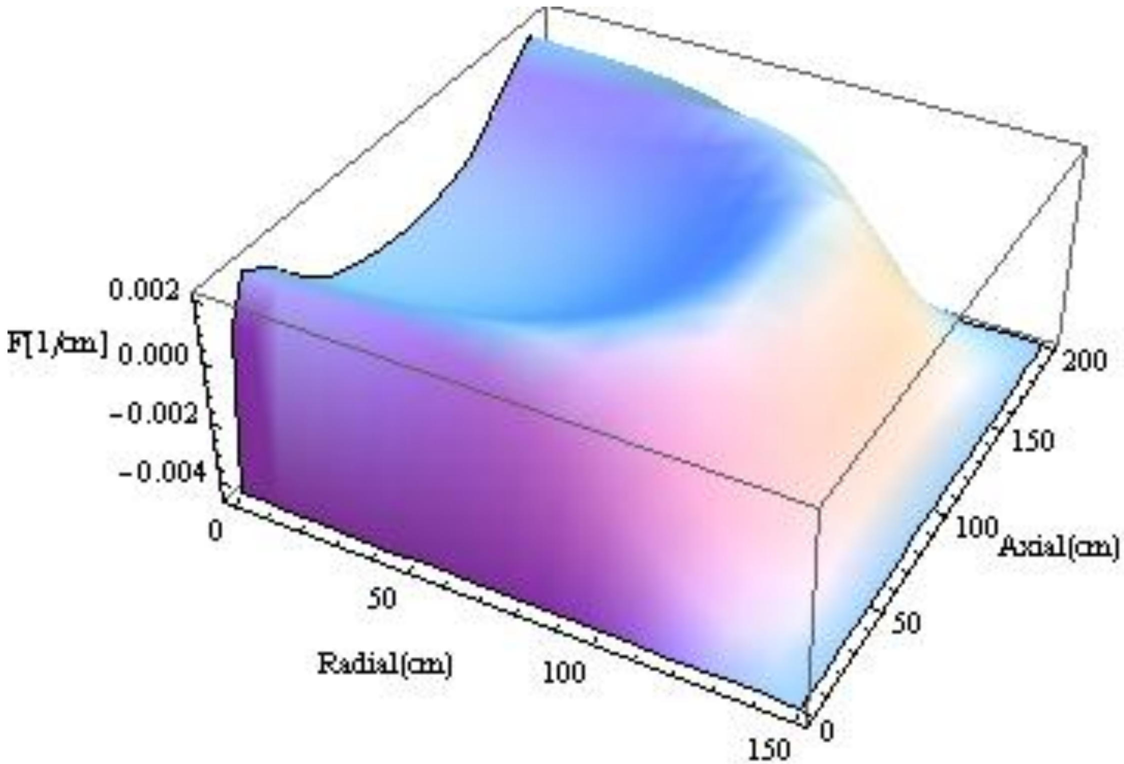
Neutron Fluence



❑ Solution and Results

➢ **Radial inward fuel shuffling**

Net Neutron Generation XS $F[\psi(r,z)]$



❑ Conclusion and Outlook

- Based on the one-group diffusion model coupled with ERANOS calculated burn-up dependent XS, the radial fuel shuffling is studied by a continuous fuel drifting model.
- The metallic fuel with pure ^{238}U is taken as fresh fuel. Standing wave solutions are obtained numerically for certain eigenvalues *k*eff and associated fuel drift speeds for the inward fuel drifting.
- This paper is just a basic theoretical study for understanding the radial fuel shuffling mechanism. Further numerical investigations are intended to do by taking account of more realistic effects, such as more accurate multi-group data.
- Parametric studies, e.g. variation of $\mathbf{v_R/\phi_{max}}$ and the reactor size, should be also carried out in order to see what is minimum and maximum of burn-up.